EFFECT OF VISCOSITY OF LIQUID PHASE ON LOSS OF HYDRODYNAMIC STABILITY IN NUCLEATE BULK BOILING

V. M. Borishanskii and M. A. Gotovskii

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In the operation of high-pressure boilers the cooling conditions may be impaired by loss of stability of nucleate boiling at the vapor-generating heating surface (so-called boiling crisis).

S. S. Kutateladze [1] has demonstrated the decisive effect of hydrodynamic stability of the two-phase boundary layer on the conditions of discontinuation of nucleate boiling. On this basis, one of the present authors [2] has proposed the following scheme for the boiling crisis phenomenon. The two-phase layer at the walls is formed by a system of liquid jets of irregular shape situated in a vapor flow. Discontinuation of nucleate boiling is regarded as a result of loss of hydrodynamic stability of motion of the phases in the boundary layer.

In the first approximation, the liquid jets may be considered cylindrical, and the thickness of the two-phase boundary layer such that the length of the continuous portion of a jet is less than the boundary layer thickness even for very small incremental oscillations. The vapor is assumed inviscid. The system of equations of motion and continuity equations for each of the two phases are examined for the case of relative motion and small oscillations of the interface. The solutions obtained are substituted into the conditions at the phase interface. The resulting algebraic equation relates the oscillation increment with wavelength [3, 4]

$$\alpha^2+\frac{2\nu k^2}{I_0(ka)}\left[I_1'(ka)-\frac{2kl}{k^2+l^2}\frac{I_1(ka)}{I_0(la)}I_1'(la)\right]\alpha=$$
$$=\frac{\sigma k}{\rho a^2}[1-k^2a^2]\frac{I_1(ka)}{I_0(ka)}\frac{l^2-k^2}{l^2+k^2}+\frac{\rho'' k^2u''^2}{\rho}\frac{k_0(ka)}{k_1(ka)}\frac{I_1(ka)}{I_0(ka)}\frac{l^2-k^2}{l^2+k^2}$$
$$\left(l^2=k^2+\frac{\alpha}{\nu}\right), \qquad (1)$$

where α is the oscillation increment, k is the wave number, ν is the kinematic viscosity of the liquid phase, ρ'' is the vapor density, ρ is the liquid density, u" is the relative velocity of the vapor in the liquid, a is the free-jet radius, σ is the surface tension at the liquid-vapor interface, and $I_i(x)$ and $K_i(x)$ are Bessel functions with imaginary argument.

We will first examine Eq. (1) without considering the influence of the viscosity of the liquid, i. e., we assume $\nu = 0$. Here, the linear term vanishes, and Eq. (1) takes the form

$$\alpha^2=\frac{\sigma k}{\rho a^2}[1-k^2a^2]\frac{I_1(ka)}{I_0(ka)}+\frac{\rho'' k^2u''^2}{\rho}\frac{K_0(ka)}{K_1(ka)}\frac{I_1(ka)}{I_0(ka)}. \qquad (2)$$

We now make quantitative estimates of the terms in the right-hand side of Eq. (2), assuming that the wave number k approaches the value $1/a$. We will obtain estimates for water boiling at atmospheric pressure. The jet diameter is much smaller than that of a vapor bubble at separation, which can be estimated from the formula [5].

$$d=0.0204\theta\left(\frac{\sigma}{g(\rho-\rho'')}\right)^{1/2}, \qquad (3)$$

where θ is the contact angle in degrees. It is safe to take $a \sim 1$ mm.

The critical load q_* for water boiling at atmospheric pressure may be taken as equal to 10^6 kcal/m$^2\cdot$ hr. The specific heat of vaporization r is ~500 kcal/kg. and the vapor density ρ'' is ~10^{-3} deg/cm^3. This yields the following estimates:

$$\sigma\sim 60\ \text{dyne/cm}, \qquad a\sim 10^{-1}\ \text{cm}, \qquad \rho\sim 1\ \text{g/cm}^3,$$
$$\frac{\rho'' k^2u''^2}{\rho}\sim 10^3, \qquad u''\frac{q_*}{r\rho''}\sim 100\ \text{cm/sec}, \qquad \frac{\sigma k}{\rho a^2}\sim\frac{60}{1\cdot 10^{-3}}\sim 6\cdot 10^4.$$

Hence the coefficient of the first term of the right-hand side of (2) is much larger than that of the second term.

We will determine the value of k at which α vanishes in the form $k = (1 + m)/a$, where m is less than unity by at least one order of magnitude. Substituting this value of k into (2) and neglecting terms of the second degree in m, we get

$$\alpha_*^2=0=-\frac{\sigma m}{\rho a^3}+\frac{\rho''(1+2m)u''k_0(1+m)}{\rho a^2k_1(1+m)}. \qquad (4)$$

Neglecting m as small in comparison with unity in the arguments of the Bessel functions and solving (4) for u", we get

$$u_*''^2=\frac{1.2\sigma m}{\rho'' a}. \qquad (5)$$

The quantity a can be obtained as follows. Let S_1 denote the area occupied by vapor, and S_2 the area occupied by liquid. We introduce the relations $S_{12} = S_1/S_2$ and $S_{21} = S_2/S_1$. Then we may write

$$a\sim\sqrt{S_{21}}d, \quad \text{or} \quad a\approx n\sqrt{S_{21}}\left[\frac{\sigma}{g(\rho-\rho'')}\right]^{1/2}, \qquad (6)$$

where n is a numerical factor. The expression for the critical velocity is then

$$u_*''^2=\frac{1.2\sigma m\sqrt{S_{12}}}{n\rho''}\left(\frac{g(\rho-\rho'')}{\sigma}\right)^{1/2}. \qquad (7)$$

The critical heat flux is

$$q_*=r\rho''\frac{1+S_{12}\rho''/\rho}{1+S_{21}}u_*'', \qquad (8)$$

or, taking (7) into consideration,

$$q_*=\left(\frac{1.2m}{n}\right)^{1/2}\frac{S_{21}^{1/4}}{1+S_{21}}\left(1+S_{12}\frac{\rho''}{\rho}\right)\left[\frac{\sigma g(\rho-\rho'')}{\rho''^2}\right]^{1/4}. \qquad (9)$$

The dimensionless form of equation (9) is

$$F_*^{1/2}=\left(\frac{1.2m}{n}\right)^{1/2}\frac{S_{21}^{1/4}}{1+S_{21}}\left(1+S_{12}\frac{\rho''}{\rho}\right)\left(F_*=\frac{q_*^2\rho''}{[\sigma g(\rho-\rho'')]^{1/2}}\right). \qquad (10)$$

where F_* coincides with the square of the parameter k introduced by Kutateladze [5].

The values of the quantities in the right-hand side of Eq. (10) can be obtained by introducing additional assumptions on the geometry and mechanism of the process.

A formula for F_* = const and $\mu \to 0$ was obtained earlier by S. S. Kutateladze [5] from an analysis of the criteria characterizing two-phase systems.

Experiment [6] shows that for a low-viscosity liquid, the F_* number depends weakly on the state parameters, and that $F_* = 0.13$ to 0.16.

If the value $k = (1 + m)/a$ is substituted into Eq. (1), the oscillation increment will no longer equal zero. We will denote this value by α_ν.

$$\alpha_\nu^2+\frac{2\nu I_1'(1+m)}{a^2I_0(1+m)}\left\{1-\frac{[1+\alpha_\nu a^2/\nu]^{1/2}I_1(1+m)I_1'(la)}{[1+\tfrac{1}{2}\alpha_\nu a^2/\nu]I_1'(1+m)I_1(la)}\right\}\alpha_\nu=0. \qquad (11)$$

We set $\alpha_\nu = -\xi\nu k^2$ and, as usual, neglect m (as small in comparison with unity) in the argument of the Bessel function. This leads to a transcendental equation in ξ

$$\xi=1.1\left\{1-\frac{(1-\xi)^{1/2}}{1-\tfrac{1}{2}\xi}0.8\frac{I_1'[(1-\xi)^{1/2}]}{I_1[(1-\xi)^{1/2}]}\right\}. \qquad (12)$$

A graphical solution of this equation yields $\xi = 2.8$ (in the solution it is necessary to pass to Bessel functions with real argument). Hence, for $k = (1 + m)/a$, we have $\xi = -2.8$. In the first approximation we will assume that $\alpha = \alpha_0 + \alpha_\nu$, where α_0 is the value of the oscillation increment that corresponds to Eq. (2). Then, in order that

$\alpha = 0$, it is necessary to modify m somewhat. The new value of m we denote by m'. Since the effect of viscosity is small, m' will be only slightly different from m.

After similar transformations, we get

$$u_*'' = 1.1\left(\frac{m'\sigma}{\rho'' a}\right)^{1/2} + 2.8\,\frac{\nu\rho^{1/2}}{a\rho''^{1/2}}. \quad (13)$$

In accordance with (8), we now pass from u_*'' to the critical heat flux

$$q_* = r\rho''\,\frac{S_{21}^{1/4}}{1+S_{21}}\left(1+\frac{S_1}{S_2}\frac{\rho''}{\rho}\right)\left[1.1\left(\frac{m'\sigma}{\rho'' b}\right)^{1/2} + 2.8\,\frac{\nu\rho^{1/2}}{a\rho''^{1/2}}\right]. \quad (14)$$

In criterial form, taking (6) into account, we get

$$F_*^{1/2} = k_0 + CA^{-1/4}. \quad (15)$$

Here,

$$k_0 = 1.1\Phi\left(\frac{m'}{n}\right)^{1/2}, \qquad \Phi = \frac{S_{21}^{1/4}(1+S_{12}\rho''/\rho)}{1+S_{21}}$$

$$C = \frac{2.8\Phi}{n}, \quad A = \frac{g}{\nu^2}\left[\frac{\sigma}{g(\rho-\rho'')}\right]^{3/2}. \quad (16)$$

and also

$$\frac{S_1}{S_2} = \frac{\varphi_*}{1-\varphi_*}, \quad (17)$$

where φ_* is the volume vapor content of the boundary layer. Then,

$$\Phi = \varphi_*^{3/4}(1-\varphi_*^{1/4})\left(1+\frac{\varphi_*}{1-\varphi_*}\frac{\rho''}{\rho}\right). \quad (18)$$

Data obtained by I. G. Malenkov [7] and by M. A. Styrikovich and E. I. Nevstrueva [8] indicate that $\varphi_* \geq 0.8$. From (18) it follows that

$$\begin{aligned} \Phi &= 0.57\,(1+4\rho''/\rho) && \text{for } \varphi = 0.8, \\ \Phi &= 0.316\,(1+99\rho''/\rho) && \text{for } \varphi_* = 0.99. \end{aligned} \quad (19)$$

It may be seen that for $\rho'' \ll \rho$ the solution for the coefficient C is only slightly sensitive to φ_*.

Substituting the empirical value $k_0 = 0.13$ into these formulas and setting $n = 1$, we find that

$$\{0.88 < C < 1.6, \quad 0.05 < m' < 0.17\} \quad \text{for} \quad \rho'' \ll \rho. \quad (20)$$

To the more probable value $\varphi_* < 0.9$ there in fact corresponds the value $m \ll 1$.

We have thus shown that the examined model of heat-transfer crisis in bulk boiling of a saturated liquid leads to results that are both qualitatively and quantitatively close to experimental data and are satisfactorily described by the semiempirical formula [2]

$$F_*^{1/2} = 0.13 + 4A^{-0.4}. \quad (21)$$

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